Studying the Sivers function by model calculations

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Abstract.
A formalism is presented to evaluate the Sivers function in constituent quark models. A non-relativistic reduction of the scheme is performed and applied to the Isgur-Karl model. The sign for the u and d flavor contributions that we obtained turns out to be opposite. The Burkardt Sum Rule is fulfilled to a large extent. After the estimate of the QCD evolution of the results from the momentum scale of the model to the experimental one, a reasonable agreement with the available data is obtained.

The partonic structure of transversely polarized nucleons is still an open problem[1]. Semi-inclusive deep inelastic scattering (SIDIS) is one of the proposed processes to access the parton distributions (PDs) of transversely polarized hadrons. SIDIS of unpolarized electrons off a transversely polarized target shows "single spin asymmetries" (SSAs) [2], due to two physical mechanisms, whose contributions can be distinguished [3, 4, 5], i.e. the Collins[2] and the Sivers[6] mechanisms. The Sivers mechanism leads to a SSA which is the product of the unpolarized fragmentation function with the Sivers PD. The latter describes the number density of unpolarized quarks in a transversely polarized target: it is a time-reversal odd, Transverse Momentum Dependent (TMD) PD. From the existence of leading-twist Final State Interactions (FSI) [7, 8], a non-vanishing Sivers function has been explained as generated by the gauge link in the definition of TMDs [9, 10, 11], whose contribution does not vanish in the light-cone gauge, as happens for the standard PD functions. Different parameterizations of the available SIDIS data have been published [12, 13, 14], still with large error bars. Since a calculation from first principles in QCD is not yet possible, several model evaluations have been performed, e.g. in a quark-diquark model [7, 15]; in the MIT bag model [16]; in a light-cone model [17].

We here describe a Constituent Quark Model (CQM) calculation of the Sivers function [18]. CQM calculations of PDs are based on a two steps procedure[19]. First, the matrix element of the proper operator is evaluated using the wave functions of the model; then, a low momentum scale, μ_0^2, is ascribed to the model calculation and QCD evolution is used to evolve the observable calculated in this low energy scale to the scale of DIS experiments. Such procedure has proven successful in describing the gross features
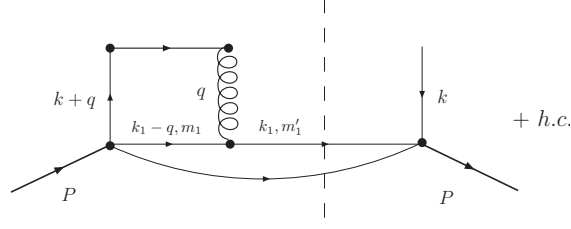
FIGURE 1. The contributions to the Sivers function in the present approach.

of PDs and GPDs[22], by using different CQMs, e.g. the Isgur-Karl (IK) model [20]. Besides the fact that it successfully reproduces the low-energy properties of the nucleon, the IK model contains the one-gluon-exchange (OGE) mechanism[21]. In the present calculation, the leading twist contribution to the FSI has to be taken into account. We, here, consider the leading, OGE, order, which is natural in the IK model. The other approximations in our approach are that, first, only the valence quark sector is investigated; second, that the resulting interaction is obtained through a non-relativistic (NR) reduction of the relevant operator, according to the philosophy of constituent quark models [21]. The Sivers function (Fig. 1) for a proton polarized along the y axis and for the quark of flavor $\mathscr{Q}$ takes the form

$$f_{1T}^{\perp \mathscr{Q}}(x, k_T) = \Im \left\{ -ig^2 \frac{M^2}{k_x} \int d\vec{k}_1 d\vec{k}_3 \frac{d^2 \vec{q}_T}{(2\pi)^2} \delta(k_3^+ - xP^+) \delta(\vec{k}_{3T} + \vec{q}_T - \vec{k}_T) \mathscr{M}^{\mathscr{Q}} \right\} \quad (1)$$

where g is the strong coupling constant, M the proton mass, and

$$\begin{aligned}
\mathscr{M}^{u(d)} = \sum_{m_1, m_1', m_3, m_3'} & \Phi_{sf, S_z=1}^{\dagger} \left(\vec{k}_3, m_3; \vec{k}_1, m_1; \vec{P} - \vec{k}_3 - \vec{k}_1, m_n \right) \\
& \frac{1 \pm \tau_3(3)}{2} V_{NR}(\vec{k}_1, \vec{k}_3, \vec{q}) \\
& \Phi_{sf, S_z=-1} \left(\vec{k}_3 + \vec{q}, m_3'; \vec{k}_1 - \vec{q}, m_1'; \vec{P} - \vec{k}_3 - \vec{k}_1, m_n \right) .
\end{aligned} \quad (2)$$

Using the spin-flavor wave function of the proton in momentum space, Φ_{sf}, corresponding to a given CQM, the Sivers function, Eq. (1), can be evaluated. From Eq. (2), one notices that the helicity conserving part of the global interaction does not contribute to the Sivers function. Besides, in an extreme NR limit, it turns out to be identically zero: In our scheme, it is precisely the interference of the lower and upper components in the four-spinors of the free quark states which leads to a non-vanishing Sivers function. This holds even from the component with $l = 0$ of the target wave function. While, in other approaches[16], these interference terms arise due to the wave function, they are produced here by the interaction.

The above-described formalism is now applied to the IK model. To evaluate numerically Eq. (1), g (i.e. $\alpha_s(Q^2)$) has to be fixed. The prescription[19] is used to fix μ_0^2, according to the amount of momentum carried by the valence quarks in the model. Here, assuming that all the gluons and sea pairs in the proton are produced perturbatively according to NLO evolution equations, in order to have $\simeq 55\%$ of the momentum
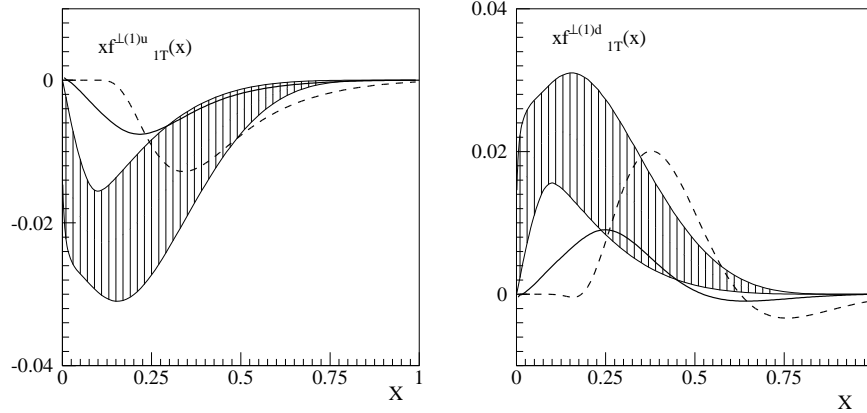
FIGURE 2. Left (right): the quantity $f_{1T}^{\perp(1)u(d)}(x)$, Eq. (3). Dashed curve: IK at μ_0^2. Full curve: the evolved distribution at NLO. Patterned area: parameterization by[13] (see text).

carried by the valence quarks at a scale of 0.34 GeV^2 one finds that $\mu_0^2 \simeq 0.1$ GeV^2 if $\Lambda_{QCD}^{NLO} \simeq 0.24$ GeV. This yields $\alpha_s(\mu_0^2)/(4\pi) \simeq 0.13$ [19]. The results of the present approach for the first moments of the Sivers function, defined as

$$f_{1T}^{\perp(1)\mathscr{Q}}(x) = \int d^2\vec{k}_T \frac{k_T^2}{2M^2} f_{1T}^{\perp\mathscr{Q}}(x,k_T) \,, \tag{3}$$

are given by the dashed curves in Fig. 2. They are compared with a parameterization of the HERMES data, taken at $Q^2 = 2.5$ GeV^2 : The patterned area represents the $1-\sigma$ range of the best fit proposed in Ref. [13]. The magnitude of the results is close to that of the data, although they have a different shape: the maximum (minimum) is predicted at larger values of x. Actually μ_0^2 is much lower, $Q^2 = 2.5$ GeV^2. A proper comparison requires QCD evolution of TMDPDs, what is, to large extent, unknown. We nevertheless perform a NLO evolution of the model results assuming, for $f_{1T}^{\perp(1)\mathscr{Q}}(x)$, the same anomalous dimensions of the unpolarized PDFs. From the final result (full curve in Fig. 2), one can see that the agreement with data improves dramatically and the trend is reasonably reproduced at least for $x \geq 0.2$. Although the performed evolution is not exact, the procedure highlights the necessity of evolving the model results to the experiment scale and it suggests that the present results could be consistent with data, still affected by large errors.

Properties of the Sivers function can be inferred from general principles. The Burkardt Sum Rule (BSR) [23] states that, for a proton polarized in the positive y direction, $\sum_{\mathscr{Q}=u,d}\langle k_x^{\mathscr{Q}}\rangle = 0$ with

$$\langle k_x^{\mathscr{Q}}\rangle = -\int_0^1 dx \int d\vec{k}_T \frac{k_x^2}{M} f_{1T}^{\perp\mathscr{Q}}(x,k_T) \,, \tag{4}$$

and must be satisfied at any scale. Within our scheme, at the scale of the model, it is found $\langle k_x^u\rangle = 10.85$ MeV, $\langle k_x^d\rangle = -11.25$ MeV and, in order to have an estimate of the quality of the agreement of our results with the sum rule, we define the ratio

$r = |\langle k_x^d \rangle + \langle k_x^u \rangle| / |\langle k_x^d \rangle - \langle k_x^u \rangle|$ obtaining $r \simeq 0.02$, so that we can say that our calculation fulfills the BSR to a precision of a few percent. One should notice that the agreement which is found is better than that found in other model calculations [15, 16], especially for what concerns the fulfillment of the Burkardt Sum Rule. However, in a recent work [24], we have shown the encouraging result that the calculation in the bag model satisfies the Burkardt sum rule at a 5% level.

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